A NEW METHOD FOR MEASURING THE PROPERTIES OF DIELECTRIC SUBSTRATE

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ABSTRACT

The TE_{01} mode in a cylindrical waveguide at a frequency below cutoff is used to probe a ceramic dielectric substrate located on the central plane between input and output coupling loops. Maximum transmission occurs at a frequency determined by the waveguide radius, the substrate thickness and the dielectric constant. The dielectric constant and loss tangent are obtained from the resonant frequency and the absorption bandwidth. The measurement is insensitive to the position of the subtrate in the gap between waveguide sections, and no intimate contact is required.

I. INTRODUCTION

The dielectric constant and loss tangent of ceramic substrates are data essential to the design of microstrip and integrated microwave circuits. Accordingly, accurate measurement methods are required by the circuit designer and the subtrate manufacturer as well. For both, the simplicity and convenience of the method are important, but the user and producer may judge these merits by different criteria.

The methods proposed in the literature require intimate contact between the dielectric and a circuit. The contact may be achieved either by metalization or by pressing a circuit on a soft substrate against the ceramic [1,2]. With metalization of two or all six sides of the substrate, a cavity resonator is formed [3,4,5], and the required data can be calculated from frequency and Q measurements. Other procedures, dependent on metalization, derive their results from measurements on specially fabricated circuits or from circuits such as microstrip lines [6] that may be provided by the manufacturer.

The merits of these methods are distributed between convenience and accuracy. When metalization is required, convenience for the user is dependent largely on the metalization pattern that is supplied by the manufacturer. For the producer of unmetalized substrates, metalization is a costly and inconvenient requirement. Since it is an irreversible process that wastes substrates, it is suited only for sampling a production run. The pressure contact with the circuit [1,2] is the method more suited for production testing, although it requires calibration with a standard substrate.

An alternative to the methods cited is one which does not depend on intimate contact to the dielectric. The basic idea is to use the TE_{01} circular waveguide mode to probe the substrate. In the fixture illustrated in Fig. 1, maximum transmission occurs at an even mode resonance of the cylindrical portion of the substrate that is exposed to the input fields. The frequency is close to that of the $TE_{01\delta}$ mode of a dielectric resonator of diameter 2a and height 2d.

The frequency is below cutoff in the circular waveguide adjacent to the substrate. In the radial waveguide formed by the gap outside the radius a, the fields are rapidly evanescent. Since there are no electric field lines normal to the surface of the substrate, intimate contact to the two surfaces of the gap is not required.

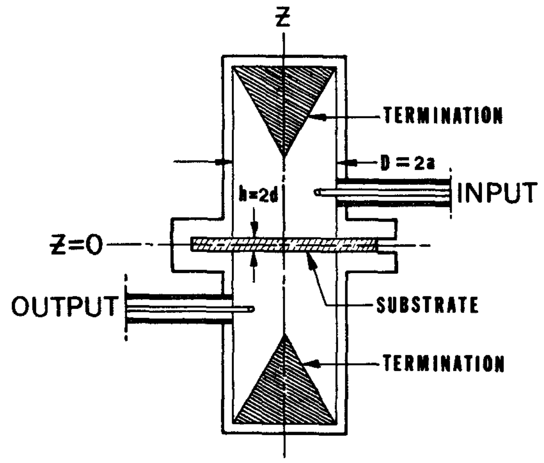

Fig. 1. Schematic cross section of substrate tester.

II. ANALYSIS OF OPERATION

Modes that propagate or are evanescent in both the empty portions of the waveguide and in the dielectric need not be considered; the terminations at the ends eliminate any narrow band resonance phenomena. It is assumed as well that a mode that is evanescent in the waveguide but propagates in the dielectric, is evanescent in the radial waveguide in the gap beyond the radius a. This justifies the use of the conducting wall boundary condition at $r = a$ in the dielectric. The error that follows from this approximation can be estimated by a perturbation calculation.

The boundary conditions at the dielectric surfaces lead to the eigenvalue equations

$$(\theta \sin\theta - \gamma \cos\theta)(\theta \cos\theta + \gamma \sin\theta) = 0 \quad (1)$$

for TE modes, and

$$(\theta \sin - K\gamma \cos\theta)(\theta \cos + K\gamma \sin\theta) = 0 \quad (2)$$

for TM modes. The notation is defined as follows:

$$\theta^2 = K\theta_o^2 - \theta_c^2 \; ; \; K = (\varepsilon'/\varepsilon_o) \; , \quad (3)$$

$$\gamma^2 = \theta_c^2 - \theta_o^2 \; , \quad (4)$$

$$\theta_c = (x_{\ell m} d/a) \; , \quad (5)$$

$$\theta_o = (\omega d/c) \; . \quad (6)$$

For a $TE_{\ell m}$ mode, $x_{\ell m}$ is the m'th zero of the derivative of J_ℓ, and for a $TM_{\ell m}$ mode it is the m'th zero of J_ℓ. If the assumptions are correct, a resonance exists such that (4) is positive, and (1) or (2) has a solution for θ in terms of γ. Then, the dielectric constant K can be calculated from (3).

In both Eqs. (1) and (2), the first term is zero for even modes, and the second vanishes for odd modes. For the reasonable constraints of $K < 100$ and $(d/a) < 0.5$, there are no odd mode solutions. The effect of the substrate on TM modes is relatively small, and the even mode TM resonant frequencies are close to their cutoff frequencies. For the TE_{01} and TM_{11} modes, which have the same cutoff frequency, the substrate produces a wide separation between the two even resonances.

The modes one expects to observe in order of increasing frequency are TE_{11}, TE_{21}, TE_{01}, TE_{31}, ... With small coupling loops in the transverse plane, coupling to the TM modes is very weak. The resonances of the first four TE modes are well separated and easily identified. All these frequencies can be used to calculate K, but only the TE_{01} mode gives reliable results. The relation between K and this frequency is shown by the curves of Fig. 2.

At resonance of the TE_{01} mode, the reciprocal of the unloaded Q is

$$(1/Q_o) \equiv D = [(\varepsilon''/\varepsilon') + (\delta/a)(\theta_c/\theta_o)^2 U]/[1+U] \quad (7)$$

where δ is the skin depth, $(\varepsilon' - j\varepsilon'')$ is the complex permittivity of the substrate, and

$$KU = [\theta \cos^3\theta]/[(\theta + \sin\theta \cos\theta) \sin\theta]. \quad (8)$$

The quantity U is the ratio of the electric energy in the waveguide to that in the substrate. As shown in Fig. 2, it increases rapidly as the cutoff frequency is approached and K decreases. It is insensitive to (d/a) over its practical range. The contribution to D from conductor losses, represented by the second term in the numerator of (7), cannot be made negligibly small, and it cannot be measured at the correct frequency in any simple way. Calculations of that term (Fig. 3), using the bulk conductivity of brass, show that it increases more or less as the root of the cutoff frequency.

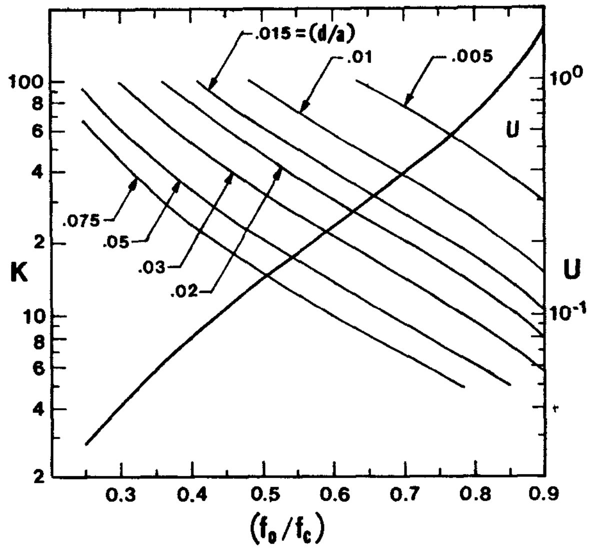

Fig. 2. Curves of K vs. (f_o/f_c) of the TE_{01} mode.

Three sources of error in the determination of K are uncertainties in the measurement of a, d, and f. These are approximated by the following expressions

$$(\Delta K/K)_a = -2(\theta_c/\theta_o)^2(U + 1/K)(\Delta a/a), \quad (9)$$

$$(\Delta K/k)_d = -\frac{2}{K}\left(\frac{\theta}{\theta_o}\right)^2 \frac{1}{1+\sin^2\theta/2\theta}\left(\frac{\Delta d}{d}\right), \quad (10)$$

$$(\Delta K/K)_{\theta_o} = -2(1 + U)(\Delta\theta_o/\theta_o). \quad (11)$$

All these error coefficients are of the order (1-5). The largest is likely to be the frequency coefficient, but the frequency can be most accurately measured.

A fourth source of error is the assumption that the electric field in the substrate is zero at r=a. This can be estimated as a frequency perturbation [7] produced by removing the stored energy of the evanescent radial modes in the space r>a. The approximate result is

$$(\Delta K/K) = (4/\pi K)(\theta_c/\theta_o)^2[1 - K(2\theta_o/\pi)^2]^{-1/2}(d/a). \quad (12)$$

The approximation is made by truncating the Fourier expansion in evanescent modes and equating the remaining coefficient to its asymptotic form. The validity depends on how small (d/a) is. For practical parameter values, $K(d/a) \sim 0.25$, and an overestimate of (12) is

$$(\Delta K/K) \sim 10\ (d/a)^2. \quad (13)$$

Errors from this source can be kept below one percent unless (d/a) and K are unusually large.

With several testers of differing radia, the TE_{01} mode can be measured over some band of frequencies. The maximum radius must be somewhat less than the width of the substrate. The minimum radius is determined by available instrumentation and the maximum permissible value of (d/a). Figure 2 suggest that it should be possible to span up to one octave.

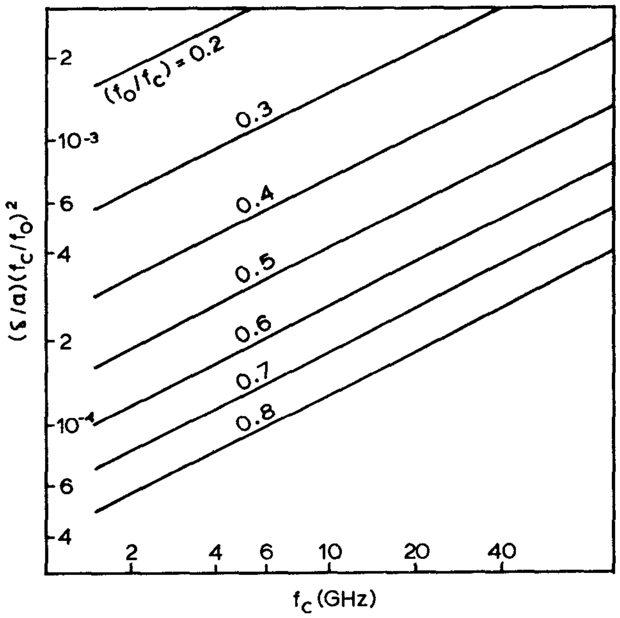

Fig. 3. Plot of $(\delta/a)(f_c/f_o)^2$ vs. f_c for brass waveguide.

III. EXPERIMENTAL RESULTS

Many resonances can be observed as maxima in the transmission through the test substrate. The lower modes can be identified by assuming a mode index, and assessing that assumption by the corresponding calculated value of the dielectric constant. If the assumption is false, the calculation yields a dielectric constant far removed from the expected value. The modes thus identified are TE_{11}, TE_{21}, TE_{01}, TE_{31}, TE_{41}, TE_{51} and TE_{02}. All modes are even with respect to z; no TM modes have been identified.

The calculations of K from these resonant frequencies give values that are low for all but the TE_{01} and TE_{02} modes. These are also distinguished by the insensitivity of the frequency to movement of the substrate. The substrate can fit loosely in the holder, but neither movement in the z-direction nor in the transverse direction has a significant effect. Although the TE_{02} mode gives good data, it is too close in frequency to other modes to allow certain identification. The TE_{01} mode is always well separated and identifiable as the third in the sequence.

In order to assess the error that comes from the extension of the substrate beyond the radius of the waveguide (gap effect), two measurements were made on a substrate of nominal K-value of 38 and thickness of 0.0385". For the first, the substrate was square (2" × 2") extending outside the 1.5" waveguide diameter. For the second, a circle was cut from the center to fit inside the tester. The results of these measurements are shown in Table I. The gap produces an apparent increase in the value of K, as is to be expected from the inductive property of the gap fields. The fractional error in K is as close to that predicted by Eq. (12) as can be expected from truncated calculations. Moreover, the somewhat loose fit of the circular substrate into the waveguide tends to increase the frequency and lower the calculated

TABLE I. Gap Effect

Dimensions	Calculated K	ΔK/K
h = .0980cm	28.1 with gap	.005
D = 3.801cm	37.9 w/o gap	.003 by Eq.(12)

The results of measurements on a variety of substrate materials and sizes are shown in Table II. Since only the TE_{01} mode is used, the frequency of measurement is increased by decreasing the tester diameter. The resulting increase in the ratio of (d/a) also increases the gap effect and the apparent dielectric constant. The bottom line is the calibration at 1 MHz obtained by a bridge measurement. For this, the substrate is metalized and a circular electrode is formed by etching a circular gap.

The gap error is illustrated in the first and last columns by the values of K (designated K*) that are corrected with the use of Eq. (12). The correction is less than one percent except for the entry in the last row, last column.

A positive error in the thickness measurement produces a positive error in the K-calculated from the capacitance data, but the error from the frequency calculation is negative. Since both error coefficients are of order one, the discrepancy between the calculated K values is approximately twice the error in the thickness measurement. If the dielectric constant is strictly independent on frequency, the thickness error can be reduced by taking the geometric mean of the tester and bridge values. This procedure applied to column one gives $\langle K \rangle = 9.895 \pm .005$ as the corrected average.

The error in the loss tangent that results from uncertainty in the effective conductivity of the waveguide can be minimized by using low loss material and choosing the frequency range so that the energy ratio U << 1. Measurement and calculations from Eq. (7) for the K38 material of row one and column eight of Table II give

$$(\varepsilon''/\varepsilon') = (2.65)[1 \pm (0.11)(\Delta\sigma/\sigma)]\ 10^{-4} \qquad (14)$$

A ten percent uncertainty in conductivity has less affect on $(\varepsilon''/\varepsilon')$ than the uncertainty in the bandwidth measurement.

IV. CONCLUDING REMARKS

This method for testing substrates is at least as simple and accurate as any other procedure that has been reported in the literature. It is limited to isotropic materials, but it can be used to explore nonuniformities providing the substrate is large. Since no metalization is required and a loose fit in the tester is permissible, it can be used for production testing.

V. ACKNOWLEDGEMENTS

The contribution of David Lupfer to the bottom line of Table II and the computational assistance of James Phillips are gratefully acknowledged.

REFERENCES

[1] A. R. Gerhard, "Measuring dielectric constant of substrates for microwave applications," IEEE Trans. Microwave Theory Tech., MTT-24, pp. 485-487, July 1976.

[2] J. Wolf, Private communication, Materials Research Corp., Orangeburg, NY 10962.

[3] L. S. Napoli and J. J. Hughes, "A simple technique for the accurate determination of the microwave dielectric constant for microwave integrated substrates," IEEE Trans. Microwave Theory Tech., MTT-19, pp. 664-665, July 1971.

[4] H. F. Lenzing, "Measurement of dielectric constant of ceramic substrates at microwave frequencies," American Ceramic Soc., Bull. vol. 51, p. 361, May 10, 1972.

[5] J. Q. Howell, "A quick accurate method to measure the dielectric constant of microwave integrated-circuit substrates." IEEE Trans. Microwave Theory Tech., MTT-21, pp. 142-43, March 1973.

[6] P. M. Pannell and B. W. Jervis, "Two simple methods for the measurement of the dielectric properties of low-loss microstrip substrates," IEEE Trans. Microwave Theory Tech., MTT-29, pp. 383-386, April 1981.

[7] J. C. Slater, Microwave Electronics, pp. 81-83, D. Van Nostrand Company, Inc., Princeton, NJ, 1950.

Table II. Measured values of K for various materials and thicknesses.

Material	K10 P125			K38 NPO						K70 NPO		
Thickness	0.0632(cm)			0.02336		0.0648		0.1282		0.1004		
Diameter	f(GHz)	K	K*	f	K	f	K	f	K	f	K	K*
3.82(cm)	8.59	9.83	9.80	7.83	38.1	5.63	38.3	4.29	38.3	3.65	68.1	67.7
3.21	9.91	9.86	9.82	8.88	38.4	6.26	37.9	4.75	38.3	4.03	68.1	67.7
2.60	11.7	9.82	9.78	10.29	38.5	7.10	38.4	6.30	38.5	4.54	68.2	67.8
1.38	14.3	9.84	9.77	12.31	38.5	8.32	38.5	7.10	38.4	5.29	68.3	67.5
Bridge	1(MHz)	10.00						1(MHz)	38.1	1(MHz)	67.3	